### 4.1 Abstraction and automation

#### 4.1.1 Problem solving

**Developing and checking solutions to simple logic problems**

It is well-known that it snows only if it is cold. “It snows only if it is cold” is called a **proposition**.

A proposition can be defined as a statement which can be **either true or false**.

For example, “Today is Friday” is a proposition which is **true or false**. If today is Friday then it is true, if it isn’t Friday today it is false.

The following are alternative phrasings of the proposition “It snows only if it is cold”:

- “If it snows, then it is cold”
- “If it is snowing, it is cold”
- “If it snows then it is cold”
- “If it is snowing then it is cold”
- “A necessary condition for it to snow is it must be cold”
- “Being cold is necessary for it to be snowing”.

- “It is cold when it snows”
- “It is cold whenever it snows”
- “It is cold if it snows”
- “It is snowing implies it is cold”
- “It is cold follows from it is snowing”
- A sufficient condition for it to be cold is it is snowing”.

**Question**

1. Write each of these statements in the form “If P, then Q”:
   (a) It snows whenever the wind blows from the north.
   (b) Getting unfit follows from not exercising enough.
   (c) Leaves on the trees turning brown implies it is autumn.
   (d) Being at a temperature of 100 °C is necessary for water to boil.
   (e) A sufficient condition for a refund is that you bought the goods in the last two weeks.

**Compound proposition**

Actually, “It snows only if it is cold” is a compound proposition because it is made up of

- two simpler propositions “It snows” and “It is cold” and
- the conditional connective, “only if”.

If we know that “It snows only if it is cold” is true then “It snows” or “It is snowing” is true only if “it is cold” is true.

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**Learning objectives:**

- Be able to develop solutions to simple logic problems.
- Be able to check solutions to simple logic problems.

**Key term**

**Proposition:**

A proposition can be defined as a statement which can be either true or false, e.g. “Today is Friday”
We use the major proposition and the minor proposition as shown below to infer or argue by deduction that if “If it is snowing then it is cold” is true and if “It is snowing” is true we may conclude that “It is cold” is true.

Major proposition: If it is snowing then it is cold
Minor proposition: It is snowing

Conclusion: Therefore, it is cold

Now consider

Major proposition: If it is snowing then it is cold
Minor proposition: It is cold

Conclusion: Therefore, it is snowing

Is this a valid conclusion?

The answer is no. It can be cold and not snowing at the same time. We are not entitled to deduce that it is snowing just because it is cold.

Given that the two propositions are true

- If it is cold, he wears a hat
- It is cold
What conclusion can you draw?

Layout your argument in the form of a major and a minor proposition followed by a conclusion as shown above.

A politician says in his manifesto:
“If I am elected, then I will lower taxes.”
The politician is elected.
Assuming that the two statements are true, what conclusion are you entitled to draw?

A politician says in his manifesto:
“If I am elected, then I will lower taxes.”
The following are true statements:
The politician is elected.
The politician doesn’t lower taxes.
Is the politician’s manifesto statement a true statement?

A student, Alice, says to another student, Ben:
“If you have a valid network password, then you can log onto the network.”
Ben replies
“I have a valid network password.”
What are the logical conditions that must apply to these two statements for the conclusion “Ben can log onto the network” to be true?

Imagine without looking, you take a card from a pack of 52 playing cards and place it face down on the table and say “If the card is the King of Diamonds, then it is red.” Anyone familiar with the colour of playing card suits (Diamonds and Hearts are red, Spades and Clubs are black) will know that this statement is true.

If you now turn over this card to reveal that it is the Queen of Spades, a black card, is it still true that “If the card is the King of Diamonds, then it is red”? The answer is, of course, yes.
Table 4.1.1 shows the truth table for the proposition “If the card is the King of Diamonds, then it is red.” The only case when this proposition is false is when “The card is the King of Diamonds” is true and “It is red” is false.

If the card is not the King of Diamonds then it could be red or black in colour but the column headed “If the card is the King of Diamonds, then it is red.” will still be true for these rows.

<table>
<thead>
<tr>
<th>The card is the King of Diamonds</th>
<th>It is red</th>
<th>If the card is the King of Diamonds, then it is red</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>True</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
<td>True</td>
</tr>
</tbody>
</table>

Table 4.1.1 Truth table for the truth of the proposition “If the card is the King of Diamonds, then it is red”

Major proposition: The card is the King of Diamonds only if it is red
Minor proposition: The card is the King of Diamonds

Conclusion: Therefore, it is a red card

Figure 4.1.1 Valid argument and a conclusion which is true

Major proposition: The card is the King of Diamonds only if it is red
Minor proposition: The card is not the King of Diamonds

Conclusion: Therefore, ?

Figure 4.1.2 Valid argument but the conclusion is inconclusive

In Figure 4.1.1, the conclusion “It is a red card” is true because both major and minor propositions are true.

In Figure 4.1.2, both major and minor propositions are true. However, we cannot draw any conclusion other than the card could be red or black because we know that cards are either red or black. But restricting ourselves to the evidence supplied, we are not entitled to use this additional knowledge.

In Figure 4.1.1, the conclusion “It is a red card” is true because both major and minor propositions are true.

In Figure 4.1.2, both major and minor propositions are true. However, we cannot draw any conclusion other than the card could be red or black because we know that cards are either red or black. But restricting ourselves to the evidence supplied, we are not entitled to use this additional knowledge.

Question

8. Draw the truth table for the proposition “If I am elected, then I will lower taxes.”

State the truth values for the row of your table which corresponds to the politician breaking his promise?

Conditional connective ⇒

If we know that “It is snowing” is true only if “It is cold” is true then using propositional variables, P and Q, where P = “It is snowing”, Q = “It is cold”, we write P ⇒ Q which reads P is true only if Q is true or P implies Q.

In the conditional statement P ⇒ Q, P is called the hypothesis (or antecedent or premise) and Q is called the conclusion (or consequence). So if the conditional statement is true and hypothesis is true then the conclusion is true.

The argument

Major proposition: If it is snowing then it is cold
Minor proposition: It is snowing

Conclusion: Therefore, it is cold

is a valid argument with a conclusion which is true.
Using propositional variables, the equivalent argument form of the above argument is

\[ P \Rightarrow Q \]

where \( \therefore \) is the symbol that denotes “therefore”.

Whenever, both \( P \) and \( P \Rightarrow Q \) are true then \( Q \) is also true.

The form of the argument that uses propositional logic variables, e.g. \( P \) and \( Q \), is called the argument form to differentiate it from the argument which uses actual statements such as “It is snowing”.

Is the conclusion always true if the argument is valid?

Consider the following valid argument

Major proposition: If computer program A compiles then it will terminate when run

Minor proposition: Computer program A compiles

Conclusion: Therefore, it will terminate when run

Even though this is a valid argument because the argument form is the conclusion is not true if the major proposition is not true.

It is a fact that some programs can get stuck in a loop and therefore will not terminate.

Question

9. Consider the following argument involving propositions:

“If the card is the King of Diamonds, then it is red.”

“The card is the King of Diamonds.”

Therefore, “The card is red.”

Write the argument form using the propositional variables, \( P \) and \( Q \), where \( P = “The card is the King of Diamonds” \) and \( Q = “It is red.” \) Is this a valid argument? Is the conclusion true?

10. The following argument consisting of two propositions and another proposition, the conclusion, is a valid argument but the conclusion is not true even though it is snowing. Why is the conclusion not true?

If it is snowing, it is warm.

It is snowing.

Therefore, it is warm.

11. The following valid argument consists of three propositions, one of which is the conclusion. The first two propositions are true. Is the conclusion true?

If it is snowing then it is cold.

It is not cold.

Therefore, it is not snowing.

12. The following valid argument consists of three propositions, one of which is the conclusion. The first two propositions are true. Is the conclusion true?

It is snowing or it is cold or it is both.

It is not snowing.

Therefore, it is cold.
Logical reasoning

A fundamental principle of logical reasoning states:

If P then Q, If Q then R, therefore, If P then R

that is, the following argument is valid

\[ P \implies Q, \quad Q \implies R \implies P \implies R \]

Consider the following statements

- If it is freezing, the streets are icy.
- If the streets are icy, accidents will happen

Applying the fundamental principle of logical reasoning we get

\[
\begin{align*}
&\text{It is freezing implies the streets are icy} \\
&\text{the streets are icy implies accidents will happen} \\
\hline
&\text{It is freezing implies accidents will happen}
\end{align*}
\]

\[
\begin{align*}
&\text{It is freezing } \implies \text{ the streets are icy} \\
&\text{the streets are icy } \implies \text{ accidents will happen} \\
\hline
&\text{It is freezing } \implies \text{ accidents will happen}
\end{align*}
\]

**Question**

13. If it snows today, then I will not go to school today. If I do not go to school today, I can catch up on my homework.

Use logical reasoning to draw a conclusion.

14. If the train arrives early then Jamin will be early for his meeting. If Jamin is early for his meeting, he can call in on his friend John. What conclusion can you draw?

15. If I don’t watch the late night film, I will go to bed early. If I go to bed early, I will wake up feeling refreshed.

What conclusion can you draw?

16. Consider the following scenario:

If Alex is allowed a TV in his bedroom, he will neglect his homework. If he neglects his homework, he will fall behind at school. If he falls behind at school, he will not progress at school. If he does not progress at school, he will need extra tuition.

Use logical reasoning to draw a conclusion.

17. If the plane arrives late and there are no taxis at the airport then Jack is late for his meeting. Jack is not late for his meeting. The plane did arrive late.

What conclusion can you draw?

18. If it is raining and Isla does not have her raincoat with her, then she will get wet. Isla is not wet. It is raining.

What conclusion can you draw.

19. Consider this rule about a set of cards:

“If a card has a vowel on one side, then it has an even number on the other side.”

Look at the cards below and answer the question which cards do I need to turn over to tell if the rule is actually true? Explain your reasoning.

U  L  8  3
From deduction to induction

Newton is well known for his three laws of motion. He is less well known for his four rules of induction. His third rule of induction is said to be at the heart of the Newtonian revolution and modern science. This rule, known as Newton’s Principle, can be expressed in modern parlance as follows:

Newton’s Principle: Whatever is true of everything before our eyes is true of everything in the universe.

Newton’s principle allows scientists to draw conclusions of a general nature from the results of experiments involving a limited set of observations and measurements.

We have learned so far about deductive reasoning, e.g.

- John Smith is a footballer
- All footballers kick a football when playing soccer

Therefore, ..........?............

The first statement is a fact about John Smith and the second is a general rule about footballers. We can infer from these two statements that John Smith kicks a football when playing soccer by applying the general rule to John.

Inductive reasoning is the inverse of deductive reasoning. In inductive reasoning we start instead with the initial and the derived facts, and look for a rule that allows the derived fact(s) to be inferred from the initial fact(s). For example,

- John Smith is a footballer
- ..........?..........?

Therefore, John Smith kicks a football when playing soccer.

One such rule is: If John Smith is a footballer, then he wears football boots when playing soccer.

This is a rule but not a very useful one because it is specific to John Smith. We use Newton’s principle to generalise this rule to make a more useful general rule:

If x is a footballer, then x wears football boots when playing soccer

where x can be anyone including people who are not footballers.

Another way of expressing this is

All footballers wear football boots when playing soccer

We would need more evidence to have faith in this general rule. This could be obtained by observing more than just one footballer playing soccer and noting over and over again that they wear football boots when playing soccer.
Note that the statement

\[ \text{If } x \text{ is a footballer, then } x \text{ wears football boots when playing soccer} \]

is neither true nor false whilst the value of \( x \) is unknown, but if \( x = \text{John Smith} \) then it becomes

\[ \text{If John Smith is a footballer, then John Smith wears football boots when playing soccer} \]

This is either true or false depending on whether John is a footballer. It is therefore a proposition.

In this logic we have statements that are not propositions until a value has been assigned to the variable(s) in the propositional function.

If we denote

\[ \text{If } x \text{ is a footballer, then } x \text{ wears football boots when playing soccer} \]

by \( P(x) \)

then \( P(\text{"John Smith"}) \) is a proposition. \( P(x) \) is said to be the value of the propositional function \( P \) at \( x \).

\[ \text{Question} \]

20. Let \( P(x) \) denote the statement “\( x > 5 \)”. What are the truth values of \( P(3) \) and \( P(7) \)?

21. The following statements are true:

   - Gerry is a computer science student.
   - All computer science students drink coffee.

   What conclusion can you draw?

22. The following statements are true:

   - Deemei is in class CS1.
   - Every student in class CS1 has learned at least one programming language.

   What conclusion can you draw?

\[ \text{Extension Question} \]

23. A boy and a girl, whose names we do not know, are sitting next to each other. One of them has fair hair, and the other dark hair. The dark-haired one says “I am a girl.” The fair-haired one says, “I am a boy.” We are told at least one of them is lying. What is your conclusion?

In this chapter you have covered:

- Developing solutions to simple logic problems.
- Checking solutions to simple logic problems.